

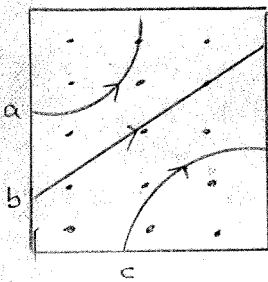
$\bar{x} = 86$   
 $s = 8$

General Instructions: For each problem your solution must be readable and your logic followable. Put a box around any numerical answers. Show your work.

**PART I - SHORT ANSWER QUESTIONS.**

Do the **five** short questions/problems. Worth 8 points each.

1. Three particles travel through a region of uniform magnetic field which is directed up out of the page as shown. They follow the paths labeled a, b, and c. Indicate whether the charge on each particle is positive, negative, or zero. Explain your reasoning.



$\vec{F} = q \vec{v} \times \vec{B}$  provides the centripetal force. If  $\vec{v} \times \vec{B}$  points to center of circle,  $q > 0$ . If opp.  $q < 0$ . If straight line  $q = 0$ .

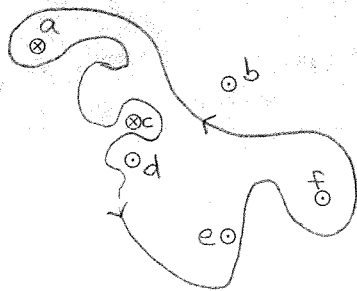
Using RHR:

$q_a$  is -

$q_b = 0$

$q_c$  is +

2. Several currents go into (indicated with an x) or come out of (indicated with a dot) the page. The currents have the values:  $i_a = 1.0$  A,  $i_b = 2.0$  A,  $i_c = 3.0$  A,  $i_d = 4.0$  A,  $i_e = 5.0$  A, and  $i_f = 6.0$  A. What is the value of integral  $\oint \vec{B} \cdot d\vec{A}$  for the loop shown, evaluated in the direction indicated by the arrow?



$$\oint \vec{B} \cdot d\vec{B} = \mu_0 i_{enc}$$

Using RHR  $i > 0$  if out of page for integration direction shown &  $i < 0$  for current into page. So looking at what's enclosed:

$$\oint \vec{B} \cdot d\vec{A} = \mu_0 (-i_a + i_d + i_c + i_f)$$

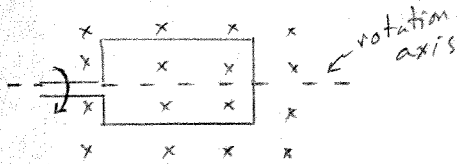
$$= \mu_0 (-1.0 \text{ A} + 4.0 \text{ A} + 5.0 \text{ A} + 6.0 \text{ A})$$

$$= \mu_0 (14.0 \text{ A})$$

$$= 1.26 \times 10^{-6} \frac{\text{Tm}}{\text{A}} (14.0 \text{ A})$$

$$\boxed{\oint \vec{B} \cdot d\vec{A} = 1.76 \times 10^{-5} \text{ Tm}}$$

3. A uniform magnetic field is directed into the page, as shown. A single loop of wire (shown at an instant when it is in the plane of the page) is caused to rotate with an angular speed of  $\omega$ . Show that a sinusoidally time varying voltage is produced.



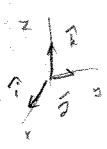
$$\begin{aligned} \mathcal{E} &= -\frac{d\Phi_B}{dt} \text{ for a single turn} \\ \Phi_B &= \vec{B} \cdot \vec{A} \text{ for a uniform field} \\ &= BA \cos \phi \\ &= BA \cos \omega t \quad \left. \begin{array}{l} \phi \text{ varies as } \omega t \end{array} \right\} \\ \mathcal{E} &= -\frac{d\Phi_B}{dt} = -BA \frac{d}{dt} \cos \omega t = +BA\omega \sin \omega t \end{aligned}$$

4. Is the way Michael Faraday viewed the relationship between his religious faith and his scientific work the same as the way Fischer (author of *God Did it, But How?*) describes as the appropriate way to relate these? Explain.

No Faraday separates areas of life  
Fischer are related

5. A straight wire carries a current of 2.50 A in the +y direction. It lies in a uniform magnetic field given by  $\vec{B} = (2.00 \text{ mT})\hat{j} + (3.00 \text{ mT})\hat{k}$ . In unit-vector notation, what is the force on a 0.500 m long segment of this wire?

$$\begin{aligned} \vec{F} &= I \vec{L} \times \vec{B} \\ &= (2.50 \text{ A})(0.500 \text{ m})\hat{j} \times [2.00 \text{ mT}\hat{j} + 3.00 \text{ mT}\hat{k}] \\ \hat{j} \times \hat{j} &= 0 \quad \hat{j} \times \hat{k} = \hat{i} \\ \text{so } \vec{F} &= (2.50 \text{ A})(0.500 \text{ m})(3.00 \text{ mT})\hat{i} \\ \vec{F} &= 3.75 \text{ mN } \hat{i} \end{aligned}$$



**Part II - Problems.**

Do all three. Worth 20 points each. Show and explain your work.

7. An RLC series circuit has  $L = 100 \overset{0.100 \text{ H}}{\underset{\text{m}}{\mu\text{H}}}$ ,  $C = 100 \overset{10.0 \text{ nF}}{\underset{\text{p}}{\text{pF}}}$ , and  $R = 10.0 \Omega$ . It is driven by a variable frequency AC generator that puts out a signal of  $(5.00 \text{ V})\sin(\omega_d t)$ .

a) At what value of  $\omega_d$  will the rms current have its maximum value?

For each of the parts below, assume that the driving frequency is twice that found in part a).

b) What rms current flows?

c) What is the phase difference between the current and driving voltage?

d) Is the circuit more capacitive or inductive? How do you know?

e) What is the average power provided by the generator?

3 pts  
 a) if  $\omega_d = \frac{1}{\sqrt{LC}}$  get max

$$\omega_d = \frac{1}{\sqrt{(0.100 \text{ H})(10.0 \times 10^{-6} \text{ F})}} = 1.0 \times 10^3 \text{ s}^{-1}$$

b) at  $\omega_d = 2.0 \times 10^3 \text{ s}^{-1}$

$$i_m = \frac{\mathcal{E}_m}{Z} = \frac{\mathcal{E}_m}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$X_L = \omega_d L = 2.0 \times 10^3 \text{ s}^{-1} \cdot 0.100 \text{ H} = 200 \Omega$$

$$X_C = \frac{1}{\omega_d C} = \frac{1}{2.0 \times 10^3 \text{ s}^{-1} (10.0 \times 10^{-6} \text{ F})} = 50.0 \Omega$$

so  $i_m = \frac{5.00 \text{ V}}{\sqrt{(10.0 \Omega)^2 + (200 \Omega - 50.0 \Omega)^2}} = 3.33 \text{ mA}$

$$i_{\text{rms}} = i_m / \sqrt{2} = \boxed{2.36 \text{ mA}}$$

c)  $\phi = \tan^{-1} \frac{X_L - X_C}{R} = \tan^{-1} \frac{200 \Omega - 50.0 \Omega}{10 \Omega} = \boxed{86.2^\circ} = 1.50 \text{ rad}$

d) inductive it lags  $\mathcal{E}$  by almost  $90^\circ$   
 (or  $X_L > X_C$ )

e)  $P = i_{\text{rms}} \mathcal{E}_{\text{rms}} \cos \phi = (2.36 \text{ mA}) \left( \frac{5.00 \text{ V}}{\sqrt{2}} \right) \cos 86.2^\circ = \boxed{5.55 \text{ mW}}$

8. A solenoid is circular in cross section and has a length of 45.0 cm, and diameter of 1.00 cm. It has a total of 600 turns and a current of 2.50 A is flowing through it.

- What is the magnetic field produced inside the solenoid?
- What is energy density stored in the magnetic field?
- What is the total energy stored in the field?
- What is the self inductance of the solenoid?
- If the current flowing through the solenoid is now reduced at the rate of 0.030 A/s, what emf is induced?

$$a) B = \mu_0 n i = 1.26 \times 10^{-6} \frac{T \cdot m}{A} (2.50 A) \left( \frac{600 \text{ turns}}{0.45 \text{ m}} \right) = \boxed{4.20 \text{ mT}}$$

$$b) u_B = \frac{1}{2\mu_0} B^2 = \frac{(4.20 \times 10^{-3} T)^2}{2(1.26 \times 10^{-6} T \cdot m/A)} = \boxed{7.00 \text{ J/m}^3}$$

$$c) U_{B \text{ tot}} = u_B \cdot \text{Vol} = \left( 7.00 \frac{\text{J}}{\text{m}^3} \right) \pi \left( \frac{0.010 \text{ m}}{2} \right)^2 (0.450 \text{ m})$$

$$= \boxed{2.47 \times 10^{-4} \text{ J}}$$

$$d) L = \mu_0 n^2 A l = 1.26 \times 10^{-6} \frac{T \cdot m}{A} \cdot \left( \frac{600}{0.45 \text{ m}} \right)^2 (\pi) \left( \frac{0.010 \text{ m}}{2} \right)^2 (0.450 \text{ m})$$

$$= 7.92 \times 10^{-5} \text{ H}$$

[ or (easier) use  $U_{B \text{ tot}} = \frac{1}{2} L i^2$  ]

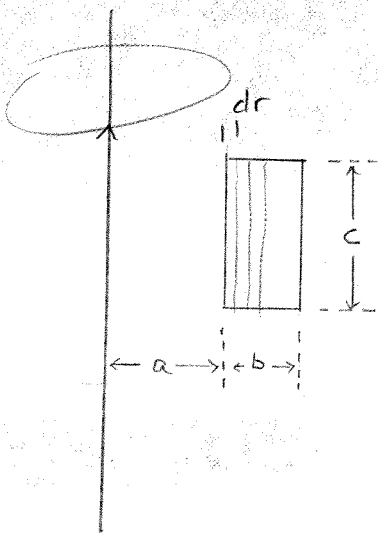
$$e) \mathcal{E} = -L \frac{di}{dt} = -(7.92 \times 10^{-5} \text{ H})(-0.030 \text{ A/s})$$

$$= \boxed{2.38 \mu\text{V}}$$

4 pts  
each

9. A long straight wire carries a time varying current  $i$  that is given by  $i = I_0 e^{-kt}$  where  $t$  is the time and  $k$  is a constant. Near this wire is a rectangular loop of wire with the dimensions shown and having resistance  $R$ .

- 3 a) At a distance  $r$  (along a perpendicular from the long wire) what is the magnetic field produced by the long wire?
- 4 b) What is the magnetic flux through the rectangular loop?
- 4 c) What current flows in the rectangular loop?
- 5 d) What is the direction of current flow in the rectangular loop? Explain.



a) From Ampere's law

$$\oint \vec{B} \cdot d\vec{r} = \mu_0 i_{enc}$$

$$B \cdot 2\pi r = \mu_0 I_0 e^{-kt}$$

$$B = \frac{\mu_0 I_0 e^{-kt}}{2\pi r}$$

b)  $\Phi = \int \vec{B} \cdot d\vec{A}$  here  $\vec{B} \parallel d\vec{A}$

so  $\Phi = \int B dA$

$dA = c dr$

so  $\Phi_B = \int_a^{a+b} \frac{\mu_0 I_0 e^{-kt}}{2\pi r} c dr$

$$\Phi_B = \frac{\mu_0 I_0 e^{-kt} c \ln\left(\frac{b+a}{a}\right)}{2\pi}$$

c)  $E_{ind} = - \frac{d\Phi}{dt} = + \frac{\mu_0 I_0 e^{-kt} k c \ln\left(\frac{b+a}{a}\right)}{2\pi}$

$$i = \frac{E_{ind}}{R} = \frac{\mu_0 I_0 e^{-kt} k c \ln\left(\frac{b+a}{a}\right)}{2\pi R}$$

d)  $\vec{B}$  from long wire points into page & decreases in time  
 Lenz's law says this change is opposed, so  $B_{induced}$  is also  
 into page clockwise current flow will accomplish this.