Bayesian Applications for Obsidian Hydration Dating
Overview

• How old is a specific obsidian artifact?
  – Radio-Carbon dating is impossible; obsidian has no carbon
  – Hydration penetration is a common surrogate for age

• Which of two artifacts is older?
  – Compare posterior distributions

• Is an artifact from a particular period?
  – Evaluate posterior distribution

The Bayesian approach easily lends itself to these questions
• Age can be inferred by shape and morphology
  – Lends itself to Prior

• More common
  – (byproduct of tool creation process)

• Cannot be easily dated
• Need model to predict age
During tool manufacture, surface of obsidian is exposed to the atmosphere.

- Water begins to slowly diffuse into the specimen.

- Rims typically vary from 1 micron (early historic period) to 30 microns (early sites in Africa).
• Many Different Volcanic Sources
• Coso Volcano
  – Arrowhead chronology and morphology well known in region
  – Isolated
  – Vast Area (can include geographical effects i.e. Temperature)
• **377 Projectile-Points (Build Data)**
  – Age: Classified into four probable ranges
    • (1-687 years old), (687-1637), (1637-4012), (4012-10,000)
  – Temperature: Based on elevation of artifact discovery
  – Hydration: Measured in microns

• **15 Radio-Carbon Pairings (Test Data)**
  – Age: Obsidian flakes discovered with artifacts that are capable of Radio-Carbon dating
  – Temperature: All discovered at same elevation leading to identical temperature estimates
  – Hydration: Measured in microns
\[ h^2 = k \cdot c \quad \text{Expanded to} \quad h^2 = \left( A e^{-E/RT} \right) c \]

Friedman and Smith 1960

Hull, 2001; Beck 1994

\( h \) = hydration rim thickness
\( A \) = source-specific constant
\( E \) = source-specific activation energy
\( R \) = universal gas constant
\( T \) = temperature
\( c \) = chronometric age (years old)

Note that hydration is the response, not age.
A stepwise regression suggests the following model:

\[ \log(h_i) = \beta_0 + \beta_1 \log(c_i) + \beta_2 \left( \frac{1}{T_i} \right) + \varepsilon_i \]

\[ \sqrt{h_i} = \beta_0 + \beta_1 \log(c_i) + \beta_2 \log(c_i)^2 + \beta_3 \left( \frac{1}{T_i} \right) + \varepsilon_i \]

**Why use root-hydration as a response?**
- Cause-effect relationship
- Data supports error structure
- Normal errors for hydration
- Discrete age observations (projectile-points)
Calibration difficulties

- Standard errors of estimates
  - Bootstrap?
- Still don’t have useful interpretations

Two roots (Green):
- Left without prediction
- Out of Luck

No roots (Red):
- Left deciding between two predictions
- Not as serious

\[ c = \exp \left[ -\beta_1 \pm \sqrt{\beta_1^2 - 4(\beta_2)(\beta_0 + \beta_3 T^{-1})} \right] \]
• Bayesian Approach
• Treat Ages of new artifacts as Missing
  – Can use certain bins as priors
• Temperature based on elevation
  – We can account for uncertainty by considering temperature as a random effect (5° margin of error)
• All explanatory variables centered to improve numerical stability
• Model

\[ h_i \mid x_{i1}, x_{i2}, \beta_0, \beta_1, \beta_2 \sim N \left( \beta_0 + \beta_1 \log^*(x_{i1}) + \beta_2 \log^*(x_{i1})^2 + \beta_3 \left( \frac{1}{x_{i2}} \right), \tau \right) \]

\[ c_i \sim \text{Uniform}(a_i, b_i) \]

\[ x_{i1} \mid c_i, \gamma_i \sim N(c_i, \gamma_i) \]

\[ x_{i2} \mid T_i, \xi \sim N(T_i, \xi) \]

• Priors

\[ \beta_0, \beta_1, \beta_2, \beta_3 \text{ iid } N(0, 1E-16) \]

\[ \tau \sim \text{Gamma}(0.0001, 0.0001) \]

• Offers better fit
• Also makes OLS calibration difficult
  • No real roots?
  • Two real roots?
• No problem for Bayes

• Informative Prior:
  Use age classification from arrowhead appearance
• Vague Prior:
  \( a=1, b=10,000 \)

• Projectile-point observed ages are intervals
• Actual ages are unobserved
• Allows actual ages to fall outside fixed intervals
• Roughly based on 10% overlap of age classes
• R, R2WinBUGS library, WinBUGS 1.4
• Data: Centered
• Iterations: 1,000,000
• Burn-in: 100,000
• Thin: 100
• Time: 24 hours
• Experience: Priceless
• With MCMC we must be sure we have convergence to assure validity of our posterior distributions

• Graphically, convergence is verified by mixing of the different colors (chains).

• Numerically, convergence is verified by the Gelman-Rubin Statistic upper bound which should be at or close to one.

• We conclude that adequate convergence has occurred.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Vague</th>
<th>Inform</th>
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<tbody>
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<td>Point Est</td>
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<td>b3</td>
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<td>deviance</td>
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</table>
### Results

<table>
<thead>
<tr>
<th>Radio-carbon</th>
<th>Posterior Age Estimates (Median)</th>
<th>Posterior Age Estimates (Mode)</th>
<th>Posterior SD</th>
<th>Prior SD*</th>
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</thead>
<tbody>
<tr>
<td>180</td>
<td>497.1 176.2%</td>
<td>327.3 45.0%</td>
<td>264.5 46.9%</td>
<td>262.4 45.8%</td>
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<tr>
<td>270</td>
<td>541.9 100.7%</td>
<td>334.4 19.3%</td>
<td>324.4 20.1%</td>
<td>231.4 -14.3%</td>
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<tr>
<td>360</td>
<td>639.0 77.5%</td>
<td>350.2 -2.8%</td>
<td>334.4 -7.1%</td>
<td>328.6 -8.7%</td>
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<tr>
<td>390</td>
<td>845.0 116.7%</td>
<td>393.2 0.8%</td>
<td>547.1 40.3%</td>
<td>451.6 15.8%</td>
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<td>410</td>
<td>848.6 107.0%</td>
<td>378.2 -8.4%</td>
<td>633.6 54.5%</td>
<td>610.9 49.0%</td>
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<tr>
<td>480</td>
<td>708.8 47.7%</td>
<td>365.3 -31.4%</td>
<td>316.3 -34.1%</td>
<td>208.7 -56.5%</td>
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<tr>
<td>860</td>
<td>1676.0 94.9%</td>
<td>1201.0 28.4%</td>
<td>1551.7 80.4%</td>
<td>1421.3 65.3%</td>
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<tr>
<td>1146</td>
<td>1801.0 57.2%</td>
<td>1199.0 4.4%</td>
<td>1608.9 40.4%</td>
<td>1402.8 22.4%</td>
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<td>1163.0 -14.4%</td>
<td>1178.5 -11.4%</td>
<td>1198.9 -9.9%</td>
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<td>1712.0 26.3%</td>
<td>1215.0 -11.6%</td>
<td>1410.2 4.0%</td>
<td>1299.3 -4.2%</td>
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<td>1410</td>
<td>2518.0 78.6%</td>
<td>1323.0 -6.6%</td>
<td>2248.5 59.5%</td>
<td>1583.6 12.3%</td>
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<tr>
<td>1713</td>
<td>2442.0 42.6%</td>
<td>2486.5 31.1%</td>
<td>2046.2 19.5%</td>
<td>1897.3 10.8%</td>
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<td>1764</td>
<td>2381.0 35.0%</td>
<td>2474.5 28.7%</td>
<td>2145.6 21.6%</td>
<td>1867.9 5.9%</td>
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<tr>
<td>6860</td>
<td>7511.5 9.5%</td>
<td>7193.0 4.6%</td>
<td>7680.2 12.0%</td>
<td>6894.0 0.5%</td>
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<td>9440</td>
<td>8364.5 -11.4%</td>
<td>8121.0 -16.2%</td>
<td>9164.6 -2.9%</td>
<td>9073.9 -3.9%</td>
</tr>
</tbody>
</table>

**RMSE:**
- 614
- 460
- 411
- 222

*Vague Prior SD = 3058

- Using the vague prior routinely overestimates age, esp. with median
- Future research will try different vague priors based on historical frequencies of different ages
- The mode performs much better when estimating older artifacts
Results

Radiocarbon Age: 390

Radiocarbon Age: 1330

Radiocarbon Age: 1713

Radiocarbon Age: 7820

Vertical Bars: Mode

Vague Posterior

Vague Prior

Informative Posterior

Informative Prior
Results

• Which of two artifacts is older?
  – Compare all 105 combinations of 15 radio-carbon posterior distributions
  – Vague: 95 of 105 had correct artifact older
    • Mean classification probability:
      \[
      \frac{1}{105} \sum_{i=1}^{15} \sum_{j=i}^{15} \hat{P}(\hat{c}_i < \hat{c}_j | c_i < c_j)
      \]
    • 80% conviction
  – Informative: 98 of 105 correct
    • 88% conviction

• Is an artifact from a particular period?
  – 14 of 15 posteriors had mode in proper bin
    • (1-687 years old), (687-1637), (1637-4012), (4012-10,000)